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# FAST AUTOMATIC PROCESSING OF RESISTIVITY SOUNDINGS *

BY

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ABSTRACT

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The difficulty to use master curves as well as classical techniques for the determination of layer distribution ($e_i$, $\rho_i$) from a resistivity sounding arises when the presumed number of layers exceeds five or six.

The principle of the method proposed here is based on the identification of the resistivity transform. This principle was recently underlined by many authors. The resistivity transform can be easily derived from the experimental data by the application of Ghosh's linear filter, and another method for deriving the filter coefficientes is suggested.

For a given theoretical resistivity transform corresponding to a given distribution of layers (thicknesses and resistivities) various criteria that measure the difference between this theoretical resistivity transform and an experimental one derived by the application of Ghosh's filter are given. A discussion of these criteria from a physical as well as a mathematical point of view follows.

The proposed method is then exposed; it is based on a gradient method. The type of gradient method used is defined and justified physically as well as with numerical examples of identified master curves. The practical use for the method and experimental confrontation of identified field curves with drill holes are given. The cost as well as memory occupation and time of execution of the program on CDC 7600 computer is estimated.

## 1. INTRODUCTION

Several authors (Koefoed 1968, Kunetz 1966, Kunetz and Rocroi 1970, Marsden 1973) have published methods for automatic interpretation of resistivity soundings. The most interesting approach seems to be that using linear filters relating (in both directions) apparent resistivities and the kernel function (Ghosh 1971).

The ideas followed in the present papers are:

— attainment of more accuracy in the filters,
— fast identification, by the gradient method, of the kernel function,
— when possible, use of a geological model.

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KUNETZ, G. and ROCROI J. P., 1970, Traitement automatique des sondages électriques, Geophys. Prosp. 18, 157-198.

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different layers, together with the boundaries imposed upon resistivities. Thicknesses were fixed to the known values within ± 0.20 m.

On fig. 4 we have represented the reconstituted and experimental curves As one can notice these curves are nearly identical, and the identification criterium is fairly good.

*Memory occupation and cost of the computer program for a CDC 7600*

Memory occupation is, for the fundamental program 10k words of 60 bits and for the actual version, 14k words of 60 bits.

Computing time for the identification of a curve is of 1.2 seconds on an average. In fact this time is dependent upon the precision desired on the criterium, and on the maximum of iterations allowed.

Both parameters are fixed by the user.

## 6. Conclusion

The method exposed, based upon identification of the resistivity transform function, has the advantage of being fast and economical. It can be applied to superficial or deep problems, with a number of layers the user chooses freely, and the possibility to impose upper and lower boundaries on the layer parameters. In this way geological data can be injected in the program. Nevertheless, in certain cases the method lacks accuracy. We are presently working on the refinement of the gradient procedure developed in 4.

Many numerical experiments were necessary before any important improvements could be achieved, but we are confident that these will never be vain.

The program is currently used on about 40% of resistivity soundings made by the Compagnie de Prospection Géophysique Française, for AB lengths usually of the order of 300 to 500 meters.

Already about 2000 soundings have been computed by the ELECTRA 02 program.

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*Example of a drilled hole fit* (fig. 4)

An electrical sounding was executed on the location of a mechanical sounding. The following table gives the resistivities, thicknesses and depths of the

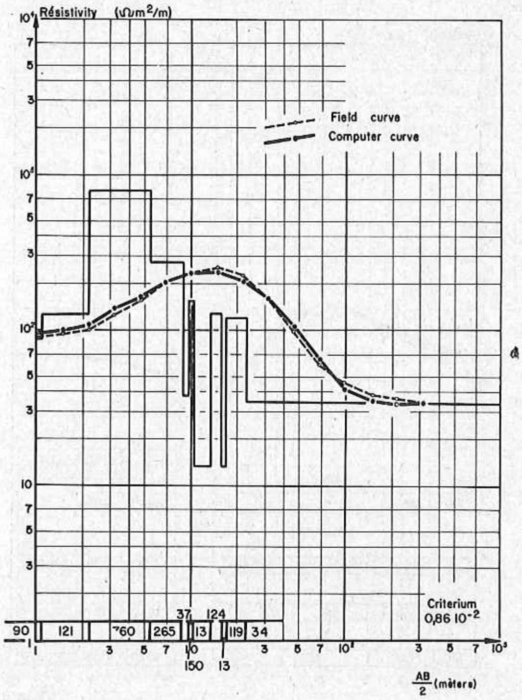


Fig. 4. Identification of resistivity sounding on drill hole.

TABLE I

*Comparison of drill hole log with resistivity interpretation*

| Drill hole log | | | | Computer results | |
|---|---|---|---|---|---|
| Depths | Section | Thickness | Limiting resistivity | Resistivity | Thickness |
| 0 - 1,10 | Clayey gravel | 1,10 m | | | 1,30 m |
| 1,10- 5,40 | Dry gravel | 4,30 | 80-1000 Ωm | 121 Ωm | 0,90 |
| | | | 500-1000 | 760 | 3,20 |
| 5,40- 8,90 | Wet gravel | 3,50 | 150- 300 | 265 | 3,30 |
| 8,90- 9,90 | Clay | 1 | 10- 50 | 37 | 1 |
| 9,90-10,80 | Gravel | 0,90 | 100- 200 | 150 | 0,80 |
| 10,80-13,40 | Clay and silt | 2,60 | 10- 70 | 13 | 2,80 |
| 13,40-16,30 | Gravel | 2,90 | 80- 250 | 124 | 3,10 |
| 16,30-17,30 | Clayey silt | 1 | 10- 70 | 13 | 0,90 |
| 17,30-23,10 | Gravel | 5,80 | 80- 250 | 119 | 5,70 |
| 23,10 | Clay | | | 34 | Total depth: 23 m |

numerical experiments have led to far better results for identified theoretical curves. A 1,5% overall error is frequently realized provided that some parameters are fixed so that no ambiguity due to the principle of equivalence remains.

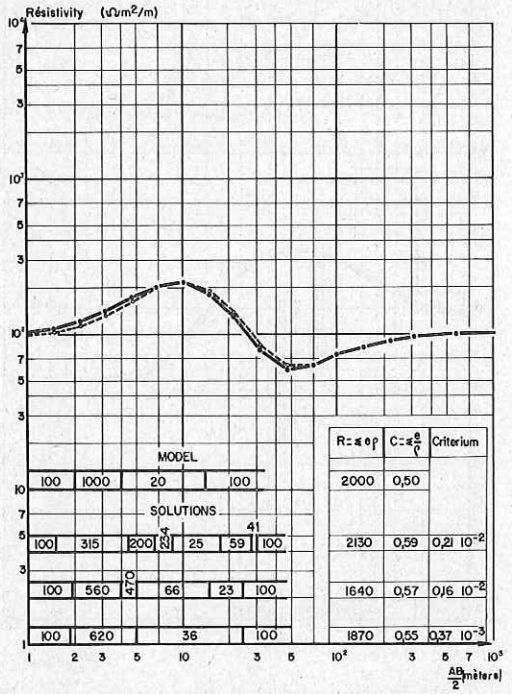


Fig. 3. Example of identification.

b) *Identification by a six layer curve*

In this case we asked the computer to carry out a six layer identification without giving any boundaries on resistivities or thicknesses. As seen on fig. 3 the given and the reconstituted curves are practically identical. We obtain $R = e_i\rho_i = 1640$ instead of 2000, and $C = e_i/\rho_i = 0{,}57$ instead of 0,50. This anomaly can be explained by the fact that the first conductor with the value of $\rho$ of 66 Ωm plays in the descending part of the curve the role of a resistor and this should be added for a part to the transverse resistance. One sees with this example the danger of applying blindly the principle of equivalence.

Analysis of Dar Zaruk curves should prove more useful.

c) *Identification with eight layers*

As seen in fig. 3 we obtain a transverse resistance of $R = 2130$ Ωm² instead of 2000 Ωm², and a conductance of 0,59 instead of 0,50.

b) *Method with a maximum degree of freedom*

In this method the number of layers is taken equal to 20 (for the present program) and no limit whatsoever is imposed upon upper and lower boundaries of thicknesses and resistivities.

The output of the computer program are:

1) a table of results: thicknesses and resistivities,
2) Dar Zaruk parameters for the identified curve,
3) the number of iterations that have been run, the value of the criterium, the module of the last computed gradient (see 4.3),
4) a diagram that represents the experimental and identified (apparent resistivity curve).

Concerning the actual criterium used (see 4), we noticed that: a criterium greater than $0{,}5.10^{-1}$ indicates a bad identification, whereas a criterium that is less than $0{,}5.10^{-2}$ indicates an excellent one.

*Numerical experiments*

We describe two numerical experiments. For more details about the possibilities of the computer program the reader is referred to (Lakshmanan, Bertrand, and Bichara 1974).

*Example 1* (fig. 3)

We have introduced the theoretical curve corresponding to the following configuration.

$\rho_1 = 100\ \Omega m$ $\quad e_1 = 2\ m$

$\rho_2 = 1000\ \Omega m$ $\quad e_2 = 2\ m$ $\quad e_2\rho_2 = 2000\ \Omega m^2$

$\rho_3 = 20\ \Omega m$ $\quad e_3 = 10\ m$ $\quad e_3/\rho_3 = 0{,}5\ \Omega^{-1}$

$\rho_4 = 1000\ \Omega m$

Many solutions that lead to this theoretical curve can be found within the boundaries:

$260 < \rho_2 < \infty$ $\quad 0 < e_2 < 7{,}7$

$0 < \rho_3 < 56$ $\quad 0 < e_3 < 2{,}8$

a) *Rigid interpretation*

Four layers with depths fixed to 5 m for the resistive layer and 25 m for the conductive layer. Resistivities are let free (no upper or lower boundaries on resistivities). The results are shown in fig. 2. As the reader can notice there is little difference between the given and the identified curves. As for the transverse resistances and conductances these are identified with 10% error; other

The calculation of the gradient of $C$ can easily by derived from the calculation of $T(\lambda, e_1, \ldots, e_{n-1}, \rho_1, \ldots, \rho_n)$ and $(\partial T/\partial e_i)$, $\partial T/\partial \rho_i)$ at each of the points $\lambda_j$, $j = 1, \ldots, k$.

This method, which could be considered as a variant of the gradient method, has been chosen after various numerical experiments. In fact, for the same reduction criterium, it resulted in the division of the number of iterations by a factor of 1.2 to 2.

A note should be made as to the initial value that should be taken for $\theta$ (see 4.1).

We took for the first iteration $\theta = 0.2\, c/\, ||\, g\, ||^2$, where:

— $c$ is the value of the criterium at the initial point
— $||\, g\, ||^2 = g^2_{\rho_2} + g^2_{e_1} + \ldots + g^2_{e_{n-1}} + g^2_{\rho_{n-1}}$ is the module of the gradient.

This corresponds to estimating that a 20% reduction of the criterium could be realized in the first iteration.

As for the computing of $T$ and its derivatives, one can apply Flathe's recurrence algorithm (Ghosh 1971).

## 5. Experimental Results

*Computer program and experimental results*

The inputs of the computer program are:

1) the values of the experimental data (value of apparent resistivities measured in the field),
2) the number of layers the user wishes to have in the solution,
3) the values of the upper and lower boundaries of thicknesses, and resistivities the user wishes to impose upon the solution, and
4) the maximum number of iterations and the criterium the user wishes to attain.

Two methods can thus be used:

a) *Fixed method*

We impose a fixed number of layers that fits the geological survey. We can then:

— fix the thicknesses and let the resistivities free in order to fit a drill hole,
— fix some of the thicknesses and let the others free, or
— impose logical upper and lower boundaries on the resistivities and let the thicknesses free

$$+ \sum_{j=1}^{K-1} \frac{((T_{j+1} - T_j) - (T(\lambda_{j+1}, \ldots) - T(\lambda_j, \ldots))^2}{| T_j(T_{j+1} - T_j) |}$$

where the significance of $T_j$ and $\lambda_j$ is clear from above. As the reader will notice such a criterium gives importance not only to the relative difference between the theoretical and experimental transform functions, but also to the difference between the derivatives of those two functions.

The gradient procedure used is basically the same as the one outlined above and differs mainly in the following points:

— in step a) introduce a value $k$ and set it to 0.
— in step d) instead of displacing the vector $\mathbf{m}$ (which here corresponds to $(e_1, \rho_2, e_2, \rho_3, \ldots, \rho_{n-1}, e_{n-1})$ of $\theta\mathbf{g}$, displace it of $\theta\mathbf{p}_k$, where

$$\mathbf{g} = (g_{e_1}, g_{\rho_2}, \ldots, g_{e_k}, g_{\rho_{k+1}}, \ldots, g_{\rho_{n-1}}, g_{e_{n-1}})$$

$$g_{e_i} = \frac{\partial C}{\partial e_i}, \quad g_{\rho_i} = \frac{\partial C}{\partial \rho_i}$$

$$\mathbf{p}_0 = (g_{e_1}, g_{\rho_2}, \ldots, g_{\rho_{n-1}}, g_{e_{n-1}})$$

$$\mathbf{p}_k = (0, 0, \ldots, g_{\rho_k}, g_{e_k}, g_{\rho_{k+1}}, 0, \ldots, 0),$$

$$\mathbf{p}_{n-1} = (0, 0, \ldots, g_{\rho_{n-1}}, g_{e_{n-1}}).$$

Furthermore if $(e_1', \rho_2', e_2', \rho_3', \rho_{n-1}', e_{n-1}')$ is the vector obtained by this displacement (represented by $\mathbf{m}_1$ in the algorithm described),
replace $(e_1', \rho_2', e_2', \rho_3', \ldots, \rho_{n-1}', e_{n-1}')$ by $(e_1, \rho_2, e_2, \rho_3, \ldots, \rho_{n-1}, e_{n-1})$
where:

$$\begin{aligned} e_i &= e_i' && \text{if} \quad e_{i,min} \leq e_i' \leq e_{i,max} \\ e_i &= e_{i,min} && \text{if} \quad e_i' < e_{i,min} \\ e_i &= e_{i,max} && \text{if} \quad e_i' > e_{i,max} \end{aligned}$$

and

$$\begin{aligned} \rho_i &= \rho_i' && \text{if} \quad \rho_{i,min} \leq \rho_i' \leq \rho_{i,max} \\ \rho_i &= \rho_{i,min} && \text{if} \quad \rho_i' < \rho_{i,min} \\ \rho_i &= \rho_{i,max} && \text{if} \quad \rho_i' > \rho_{i,max} \end{aligned}$$

— in step e) add 1 to $k$, and if $k > n - 1$, set $k$ to 0.

These modifications correspond to making a cyclical search by varying the parameters of each layer independently, and every $n$ iterations make a global search for all parameters. Furthermore, during the search, in step d), we keep the parameters within the specified intervals.

4.3. *Algorithm* (see fig. 2)

The algorithm developed is based on a gradient procedure. As explained above, a criterium that measures the difference between the experimental and

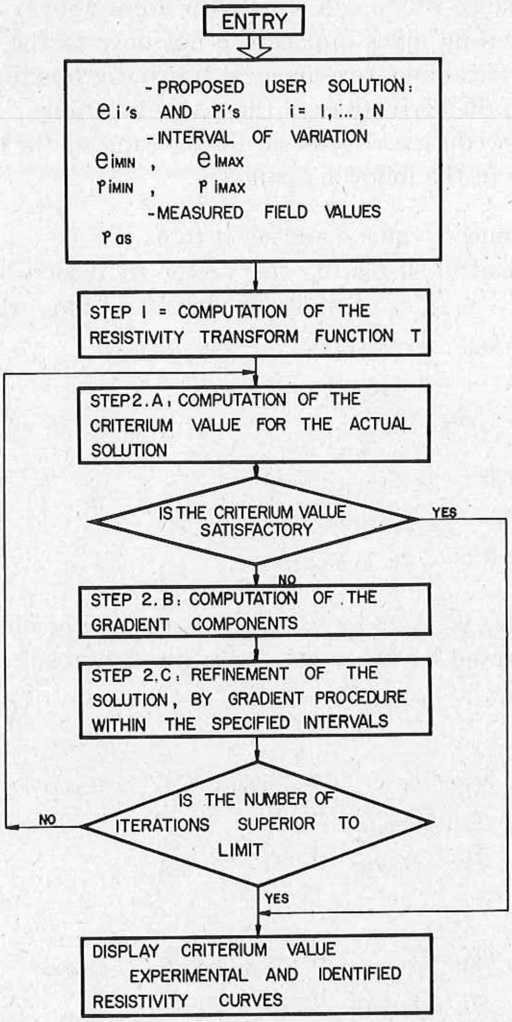


Fig. 2. Flow-chart of the computer program.

theoretical function should be defined. The criterium we have chosen after various numerical experiments is the following:

$$C(e_1, e_2, \ldots, e_{n-1}, \rho_1, \rho_2 \ldots, \rho_n) = \sum_{j=1}^{K} \frac{(T_j - T(\lambda_j, e_1, \ldots, e_{n-1}, \rho_1, \ldots, \rho_n))^2}{T_j^2}$$

If $C(\mathbf{m}_1) < C(\mathbf{m}_0)$ resume step c) replacing $\mathbf{m}_0$ by $\mathbf{m}_1$, else set $\theta = \theta/2$ and resume step d)

f) stop the calculation whenever attaining a value $| C(\mathbf{m}) | < \varepsilon$ where $\varepsilon$ represents the desired precision of identification for the criterium $C$.

For more details about gradient methods the reader is referred to Abadie (1966).

### 4.2. *Identification of the resistivity transform*

Let $T_1, T_2, \ldots, T_k$ be the values of the resistivity transform at points $\lambda_1, \ldots, \lambda_k$. We wish to obtain a theoretical function $T(\lambda, e_1, \ldots, e_{n-1}, \rho_1 \ldots, \rho_n)$ that best fits the values $T_1, \ldots, T_e, \ldots, T_k$ at points $\lambda_1, \ldots, \lambda_k$, and that fullfills the following requirements:

a) the number $n$ of layers is fixed.
b) $\rho_1$ and $\rho_n$ are given
c) the variation of the resistivities $\rho_i$ and of the thicknesses $e_i$ are confined within certain intervals, i.e.

$$\forall i, i = 2, \ldots, n-1$$
$$\rho_{i,min} \leq \rho_i \leq \rho_{i,max}, \text{ and}$$
$$\forall i, i = 1, \ldots, n-1$$
$$e_{i,min} \leq e_i \leq e_{i,max}.$$

Posing the problem in this form is important for the practical geophysician. Very often he is confronted with the problem of determining the resistivities for drill holes for which he has the thicknesses. Furthermore, the knowledge of the geology of the soil he is surveying permits him to suppose the limits of certain layers. Thus, adapting the algorithm to fulfill the requirements defined by points a), b) and c) of (4.2) above is of great help to the interpreter.

The authors wish to underline here that, if the algorithm was written to find systematically a solution with a constant number of layers and with no limitations on thicknesses or resistivities, a Dar Zaruk equivalent solution should afterwards be sought in order to adapt the interpretation to the geology.

Nevertheless, the interpreter, if he has no idea whatsoever of the resistivity and thicknesses distribution, can assign to $n$ a large value (20 as a maximum for the present computer program) and give fairly large intervals of variations to the resistivities and thicknesses. He could then interpret the Dar Zaruk curve thus obtained.

An algorithm was developped to treat the problem of identification and a programme was written in Fortran. Numerical experiments are exposed in chapter 5.

This method was also used to recalculate Ghosh's coefficients. The results were satisfactory.

## 4. Automatic Identification of Resistivity Sounding

To a given layer distribution of thicknesses $e_1, \ldots, e_i, \ldots e_n$ and resistivities $\rho_1, \ldots, \rho_j, \ldots, \rho_n, \rho_n$ being the substratum resistivity, corresponds one and only resistivity transform:

$$T(\lambda, e_1, \ldots, e_j, \ldots e_n, \rho_1, \ldots, \rho_j, \ldots \rho_n).$$

Thus, identification of an apparent resistivity function is equivalent to the identification of its associated resistivity transform, which can easily be obtained by the application of Ghosh's filter or any other filter obtained by the methods suggested in the preceding section.

### 4.1 *General discussion of function identification*

The problem of the identification of the resistivity transform $T$ is the same as the identification of any experimental function with a theoretical one $f(\lambda, \bar{m})$, where $\bar{m} = (m_1, \ldots, m_k)$ are the parameters that one wishes to determine.

There is a great variety of methods that has been developed for such an identification. A good number of these are based on gradient methods used to minimize a criterium that measures the difference between the experimental and theoretical function.

If $F_j$, $j = 1, \ldots, n$ are the values of the experimental function at the points $\lambda_j$, a largely used criterium to measure the difference between this function and a theoretical one $f(\lambda, \bar{m})$ is:

$$C(\bar{m}) = \sum_{j=1}^{n} (F_j - f(\lambda_j, \bar{m}))^2 \qquad (4.1)$$

The gradient of $C$ relative to $\mathbf{m} = (m_1, \ldots, m_k)$ is equal to:

$$\left(\frac{\partial C}{\partial m_1}, \frac{\partial C}{\partial m_2}, \ldots, \frac{\partial C}{\partial m_j}, \ldots, \frac{\partial C}{\partial m_k}\right)$$

where

$$\frac{\partial C}{\partial m_e} = -2 \sum_{j=1}^{n} \frac{\partial f(\lambda_j, m)}{\partial m_e} (F_j - f(\lambda_j, m)). \qquad (4.2)$$

The simplest gradient method used could be resumed by the following steps:

a) choose a certain value $\theta > 0$ and a logical value $\mathbf{m}_0$
b) compute $C(\mathbf{m}_0)$
c) compute the gradient $\mathbf{g}(\mathbf{m}_0) = (\partial C/\partial \mathbf{m}) \,/\, \mathbf{m} = \mathbf{m}_0$
d) compute a new value of $\mathbf{m}$, $\mathbf{m}_1 = \mathbf{m}_0 - \theta \mathbf{g}$
e) compute $C(\mathbf{m}_1)$

significance of $\Delta 1$ and $\Delta 2$ is clear from fig. 1. Thus, if one chooses $K = [\Delta 1/\Delta x] + 1$, and $L = [\Delta 2/\Delta x] + 1$ the calculation will be exact provided that we adopt for $h(K\Delta x)$ the value $\sum_{i=-\infty}^{i=-K} h(i\Delta x)$ and for $h(L\Delta x)$ the value $\sum_{i=L}^{i=+\infty} h(i\Delta x)$, and that we take for $h(j\Delta x)$ $j = -k + 1, \ldots, L$, the true value calculated in equation (3.1).

It should be remarked that, according to the preceding discussion, the number of coefficients to be considered is dependent on the domains $\Delta 1$ and $\Delta 2$ at the exterior of which the considered resistivity curve has its value practically equal to $\rho_1$ or $\rho_n$ respectively (see fig. 1).

Concerning this, numerical experiments drawn with Ghosh's "long" filter coefficients proved highly satisfactory, despite the fact that this filter was composed of only twelve elements.

The second method we suggest is simpler in its aspects but less attractive in its theoretical basis. Nevertheless, it is in part based on the assumption that, provided the number of filter elements is large enough, there should exist a linear filter that realizes the the transformation of $\rho_{as}$ into $T$ (see the above discussion).

Let us consider for example the equation:

$$(\lambda + \lambda^2) / (15 e^{\lambda}) = \int_{-\infty}^{+\infty} \frac{s^3}{(1 + s^2)^{7/2}} \frac{J_1(\lambda s)\, ds}{s} \qquad \text{(Koefoed 1968).}$$

The variable change given in 2 gives

$$(e^{-y} + e^{-2y})/(15\, e^{e^{-y}}) = \int_{-\infty}^{+\infty} \frac{e^{3x}}{(1 + e^{2x})^{7/2}} \bar{J}_1(y - x)\, dx.$$

If the Fourier spectrum of $e^{3x}/(1 + e^{2x})^{7/2}$ could be considered as practically band-limited within $]-f_c, +f_c[$, we can write with the notation of the previous paragraphs:

$$(e^{-m_0 \Delta x} + e^{-2m_0 \Delta x}) / 15\, e^{e^{-m_0 \Delta x}} \backsimeq \sum_{j=-K}^{j=L} h(j\Delta x) \frac{e^{2(m_0 - j)\Delta x})^{7/2}}{(1 + e^{2(m_0 - j)\Delta x})^{7/2}}$$

for $m_0 = -K, \ldots, 0, \ldots, L$.

This is a system of $(K + L + 1)$ linear equations with $(K + L + 1)$ linear unknowns $h(j\Delta x)$, $j = -K, \ldots, 0, \ldots, L$, which are nothing else than the filter coefficients.

We have also calculated a 29 element filter for a cut-off frequency of $f_c = 3/(\ln 10)$. We give the coefficients obtained:

$h(k)$, $k = -9, \ldots, 0, \ldots, 19$:

| | | | | | | |
|---|---|---|---|---|---|---|
| —0.0074 | 0.01339 | —0.01992 | 0.02946 | —0.0085 | —0.1163 | |
| 0.11696 | 0.2071 | 0.2192 | 0.1631 | 0.1283 | 0.0823 | 0.0638 |
| 0.0376 | 0.0313 | 0.0163 | 0.0157 | 0.00644 | 0.00837 | 0.00198 |
| 0.00484 | 0.000009 | 0.00312 | —0.0008 | 0.00224 | —0.0011 | 0.0018 |
| —0.0012 | 0.0020 | | | | | |

Before exposing the second method suggested it should be useful to make a note as to the number of coefficients that should be calculated in equation (3.1). Refer to fig. 1 for the following discussion.

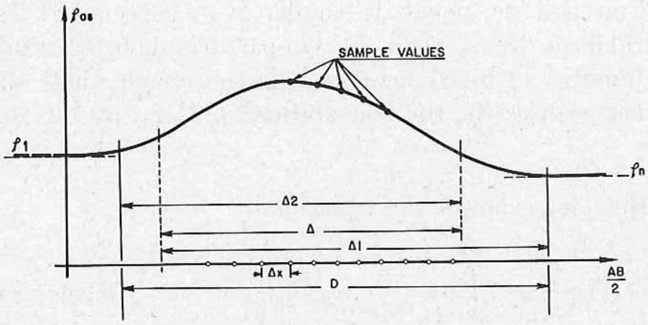


Fig. 1. Application of a linear filter to a resistivity sounding.

Let $\Delta$ be the domain in which the values have been effectively measured in the field, and $D$ be the domain at the exterior of which $\rho_{as} \backsimeq \rho_n$ (on the right) and $\rho_{as} \backsimeq \rho_1$ (on the left), and suppose we want to obtain the values of $T(y_1)$ in the entire domain $\Delta$. We will proceed as follows:

a) we take sampled points on the curve of amplitude $\Delta x$

b) to calculate the value $\overline{T}(y_0)$ at a sampled point $y_0$ we calculate

$$\overline{T}(y_0) = \sum_{m=-K}^{m=+L} \bar{\rho}_{as}(y_0 - m\Delta x)\, h(m\Delta x), \text{ where } h\,(m\Delta x)$$

is the $m$th filter coefficient and $(K + L + 1)$ is the number of coefficients considered.

Theoretically, the number of coefficients should be infinite. However, the maximum number of coefficients with a negative index that multiply a value $\rho_{as}$ different from $\rho_n$ is equal to $[\Delta 1/\Delta x]$ (where $[u]$ represents the smallest integer superior to $u$), and the maximum number of coefficients with a positive index that multiply a value $\rho_{as}$ different from $\rho_1$ is equal to $[\Delta 2/\Delta x]$ where the

We must, however, underline that the function to integrate (3.1) is rapidly oscillating when $u$ takes great values, and thus, a lot of care should be taken when the integration concerns these values (the pace should then be at least equal to half the period of oscillation).

We have recalculated Ghosh's coefficients by this method. The limits of integration taken are $-10 + k\Delta x$, $20 + k\Delta x$ with $\alpha = u - k\Delta x$ in equation (3.1).

The pace of integration we took is the following:

$$
\begin{array}{lll}
. \ p = \pi/16 & \text{for} & \alpha < -8 \\
. \ p = \pi/50 & ,, & -8 \leq \alpha < -5 \\
. \ p = \pi/100 & ,, & -5 \leq \alpha < -2 \\
. \ p = 2.10^{-3} & ,, & -2 \leq \alpha < 6 \\
. \ p = 1/e^{u - k\Delta x} & ,, & 6 \leq \alpha < 8 \\
. \ p = 2/e^{\alpha} & ,, & 8 \leq \alpha \leq 10
\end{array}
$$

where $p$ represents the pace of integration.

From the asymptotic behavior of $\overline{J}_1$

$$\overline{J}_1(-\alpha) \simeq \sqrt{\frac{2}{\pi e^{\alpha}}} \cos\left(e^{\alpha} - \frac{3\pi}{4}\right) \qquad \text{when } \alpha \to +\infty,$$

and from

$$e^{\alpha + \Delta\alpha} = e^{\alpha} + \Delta\alpha e^{\alpha} + \ldots$$

we deduce that $2\pi/e^{\alpha}$ approximately represents the asymptotic period of oscillation of function $\overline{J}_1$, when $\alpha \to +\infty$. Thus, the last pace of integration represents approximately 1/3rd of the period of oscillation. The results are nevertheless satisfactory, probably because of the little significance of the integral in the concerned interval.

The values obtained are: $h(-3)$, $h(-2)$, $h(-1)$, ..., $h(8)$ = 0.00659, —0.07828, 0.39991, 0.34916, 0.16754, 0.08586, 0.03575, 0.01977, 0.0067, 0.00514, 0.00067, 0.0018

These values, if rounded are equal to those calculated by Ghosh (1971).

We insist on the importance of the pace taken for the integration. In fact, a simple Simpson variable pace integration method, taken from our mathematical library of programs gave for $h(k)$, $k = -3, \ldots, 0, \ldots, 8$: 0.00378, —0.07908, 0.3958, 0.3487, 0.1689, 0.0891, 0.0359, 0.0175, 0.0058, 0.0073, —0.0034, 0.00123, which could be qualified as comparatively poorresults. Computing time was of 11.8 seconds on a CDC7600 for the first method, and only of 0.6 seconds for the second method.

spacing of electrodes in field work, and the effective calculation of the Fourier spectrum of some two and three layers master curves has led him to choose for $f_c$ the value $3/(2 \ln (10)$, (i.e. $\Delta x = \frac{1}{3} \ln (10)$) where ln represents the Neperian logarithm. Thus, it should be noticed that for curves of apparent resistivities $\rho_{as}$ whose spectrum extends far beyond these limits the theory is not valid.

b) Experimentally, a curve of apparent resistivity $\rho_{as}$ has a band limited spectrum contained within the limits $]-f_c, +f_c[$ if its variations are very smooth compared with any sinusoïd of frequency superior to $f_c$.

c) Due to the fact that a one layer apparent resistivity curve is a constant equal to $\rho_1$ and thus has a band limited spectrum (the well known Dirac function), and that its resistivity transform is also $\rho_1$, we can deduce from eq. (2.6) that for any filter whatever the frequency $f_c$ is:

$$\sum_{m=-\infty}^{m=+\infty} h(k\Delta x) = 1, \text{ where } \Delta x = 1/2f_c$$

## 3. Methods Suggested for the Calculation of Ghosh's Linear Filter

The first method we suggest is based on the direct calculation of the integral defined in (2.5). This calculation is necessarily a numeric calculation; therefore, the question arises as to the limit within which the calculation is to be carried out.

Setting $k = (m - m_0)$, equation (2.5) becomes

$$h(k\Delta x) = \int_{-\infty}^{+\infty} (\sin(\pi u/\Delta x) \,/\, (\pi u/\Delta x))\, \bar{J}_1(k\Delta x - u)\, du \qquad (3.1)$$

where $\Delta x = 1/(2f_c) = \frac{1}{3} \ln 10$, and $\bar{J}_1(\alpha) = J_1(e^{-\alpha})$.

Elementary properties of Bessel function $J_1$ lead us to write for $u \to -\infty$: $\bar{J}_1(k\Delta x - u) = J_1(e^{u-k\Delta x}) \simeq \frac{e^{u-k\Delta x}}{2}$, and for $u \to +\infty$; $\bar{J}_1(k\Delta x - u) \simeq \sqrt{\frac{2}{\pi e^{\alpha}}}$ $\cos\left(e^{\alpha} - \frac{3\pi}{4}\right)$, where $\alpha = u - k\Delta x$.

Thus, $\bar{J}_1(k\Delta x - u)$ is exponentially converging towards 0 when $u$ tends towards $-\infty$ or $+\infty$.

Furthermore $|\sin(\pi u/\Delta x) \,/\, (\pi u/\Delta x)|$ is also converging towards 0 when $u$ tends towards $-\infty$ or $+\infty$.

Thus, the coefficient defined by (3.1) could accurately be numerically calculated without having to take too large integration boundaries.

For example, if a precision of $0{,}5 \cdot 10^{-4}$ is desired for the values of $h(k\Delta x)$, it is sufficient to integrate the function defined by equation (3.1) between the boundaries: $-10 + k\Delta x$, $20 + k\Delta x$.

In other terms, the knowledge of $\bar{\rho}_{as}$ and $\bar{T}$ at every point $x$ or $y$ is equivalent to the knowledge of $\bar{\rho}_{as}$ and $\bar{T}$ at a set of sampled points.

From (2.3) and (2.1) we deduce:

$$\bar{T}(y_0) = \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{m=+\infty} \bar{\rho}_{as}(m\Delta x) \sin\left(\frac{\pi(x-m\Delta x)}{\Delta x}\right) \Big/ \left(\frac{\pi(x-m\Delta x)}{\Delta x}\right) \bar{J}_1(y_0-x)dx$$

$$= \sum_{m=-\infty}^{m=+\infty} \bar{\rho}_{as}(m\Delta x) \cdot \int_{-\infty}^{+\infty} \left(\sin\left(\frac{\pi u}{\Delta x}\right) \Big/ \left(\frac{\pi u}{\Delta x}\right)\right) \bar{J}_1((m_0-m)\,\Delta x - u)\,du,$$

where,

$$u = x - m\Delta x \quad \text{and,}$$
$$y_0 = m_0\Delta x, \quad \Delta x = 1/(2f_c)$$

If we set

$$h((m_0-m)\,\Delta x) = \int_{-\infty}^{+\infty} \sin(\pi u/\Delta x)/(\pi u/\Delta x)\ \bar{J}_1\,(m_0-m)\,\Delta x - u)\,du \qquad (2.5)$$

we have

$$\bar{T}(m_0\Delta x) = \sum_{m=-\infty}^{m=+\infty} \bar{\rho}_{as}\,(m\Delta x)\,h((m_0-m)\,\Delta x) \qquad (2.6)$$

The quantities $h((m_0 - m)\,\Delta x)$ represent Ghosh's filter coefficients. Ghosh has calculated them indirectly by noting that the Fourier transform of

$$h(z) = \int_{-\infty}^{+\infty} (\sin\,(\pi u/\Delta x)/(\pi u/\Delta x))\ \bar{J}_1(z-u)\,du$$

is given by

$$\mathfrak{F}(h) = \mathfrak{F}(\sin\,(\pi u/\Delta x)\,/\,(\pi u/\Delta x)) \cdot \mathfrak{F}(\bar{J}_1),$$

Since $\mathfrak{F}(\sin(\pi u/\Delta x)\,/\,(\pi u/\Delta x))$ is the well known constant band limited filter, $\mathfrak{F}(h)$ is equal to the restriction of $\mathfrak{F}(\bar{J}_1)$ on the interval $]-f_c, +f_c[$. Thus, to calculate the filter coefficients one has to calculate $\mathfrak{F}(\bar{J}_1)$ in the interval $]-f_c, +f_c[$ and, by calculating the Fourier inverse, compute the value $h(k\Delta x)$ which are the filter coefficients.

Before defining any of the methods suggested for the filter calculation, it seems useful for the comprehension of the experimental part of this paper to underline the following points:

a) the theory is entirely based on the fact that $\rho_{as}$ has a band limited spectrum between $-f_c$ and $+f_c$. Ghosh's choice of $f_c$ based upon the classical

## 2. Ghosh's Theory and Method for the Filter Determination

The apparent resistivity in the Schlumberger arrangement is given by the classical Stefanesco relation:

$$\rho_{as} = \rho_1 (1 + 2s^2 \int_0^{+\infty} B(\lambda)\, J_1(\lambda s)\, \lambda d\lambda)$$

where $B(\lambda)$ is the kernel function. The resistivity transform is defined by

$T(\lambda) = \rho_1(1 + 2B(\lambda))$, and (Ghosh 1971), we have:

$$T(\lambda) = \int_0^{+\infty} \rho_{as}(s)\, J_1(\lambda s)/s\, ds$$

Setting $s_1 = e^x\ \lambda = e^{-y}$ in the above integral one obtains:

$$T(e^{-y}) = \int_{-\infty}^{+\infty} \rho_{as}(e^x)\, J_1(e^{x-y})\, dx$$

if we define:

$$\bar{T}(y) = T(e^{-y}), \quad \bar{\rho}_{as}(x) = \rho_{as}(e^x), \quad \bar{J}_1(u) = J_1(e^{-u})$$

we have:

$$\bar{T}(y) = \int_{-\infty}^{+\infty} \bar{\rho}_{as}(x)\, \bar{J}_1(y - x)\, dx \tag{2.1}$$

thus $\bar{T}(y)$ is obtained as the convolution product of $\bar{\rho}_{as}$ and $\bar{J}_1$; the application of the Fourier transformation to both sides of equation (2.1) gives:

$$\mathfrak{F}(\bar{T}) = \mathfrak{F}(\bar{\rho}_{as}) \cdot \mathfrak{F}(\bar{J}_1) \tag{2.2}$$

where $\mathfrak{F}(g)$ denotes the Fourier transform of $g$. Ghosh's idea is then the following:

If the Fourier spectrum of $\bar{\rho}_{as}$ is band limited;

i.e. $\mathfrak{F}(\rho_{as})\,(\nu) \simeq 0$ if $\nu$ does not belong to the interval $]-f_c + f_c[$,

then we deduce from (2.2) that $\mathfrak{F}(\bar{T})$ is also band limited, and according to Shannon's theorem (Ghosh 1971) we can write:

$$\bar{\rho}_{as}(x) = \sum_{m=-\infty}^{m=+\infty} \bar{\rho}_{as}(m\Delta x) \sin\left(\frac{\pi(x - m\Delta x)}{\Delta x}\right) \Big/ \left(\frac{\pi(x - m\Delta x)}{\Delta x}\right) \tag{2.3}$$

and

$$\bar{T}(y) = \sum_{m=-\infty}^{m=+\infty} \bar{T}(m\Delta y) \sin\left(\frac{\pi(y - m\Delta y)}{\Delta y}\right) \Big/ \left(\frac{\pi(y - m\Delta y)}{\Delta y}\right) \tag{2.4}$$

where

$$\Delta x = \Delta y = 1/(2f_c)$$