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AUTOMATIC DECONVOLUTION OF GRAVIMETRIC ANOMALIES*

BY

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ABSTRACT

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Existing techniques of deconvolution of gravity anomalies are principally based on upward and downward continuation of measured fields. It can be shown that a unique set of linear filters, depending only on geometrical parameters, relates density distribution at a given depth to gravity measured on the surface. A method to compute the filter coefficients is developed. Very accurate reconstitution of theoretical models of intricate shape, prove the validity of the linear relationship. One of these sets of linear filters is applied to a field case of underground quarries.

I – LINEARITY OF THE RELATIONSHIP BETWEEN GRAVITY AND DENSITY DISTRIBUTIONS

I.1 – *LA PORTE method*:

La Porte (1963) established an iterative method for the reconstruction of an underground geological structure from the gravity field on the surface of the ground.

In the first step the field is continued upwards:

$$g\,(x, y, +z) = \frac{z}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{g(x, y, 0)}{(x^2+y^2+z^2)^{3/2}}\, dx\, dy \qquad (1)$$

Next the field is continued downward by:

$$g\,(x, y, -z) \doteq -g\,(x, y, +z) + 2g\,(x, y, 0) + z^2 \frac{\partial_g^2}{\partial_z^2}(x, y, 0) + 2 \sum_{i=2}^{\infty} \frac{z^{2i}}{(2i)!} \frac{\partial_g^{2i}\,(x, y, 0)}{\partial z^{2i}}. \qquad (2)$$

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Finally, La Porte calculates a surface density distribution at depth z from

$$\sigma(x, y, -z) = \frac{3g(x, y, -z)}{4\pi} \tag{3}$$

or a volume density distribution from

$$\delta(x, y, -z) = \frac{3g(x, y, -z)}{4\pi e}, \tag{4}$$

where e is the thickness of the layer at depth $-z$.

Equation (4) and (5) suppose that the variation of the gravity field are due to the density variations within *one layer* at depth $-z$.

The La Porte method is very interesting and justifies itself by equations (1) to (3). It has been used by us for several problems. Nevertheless, it is a costly method needing many computations (see equations (1) and (2)). Furthermore, problems due to the discretisation of equation (2) often arise. These problems were already discussed in detail in La Porte (1963).

I.2 – *Relationship between gravity and density distribution*:

From equations (1), (2), and (3) one notes that, for a given depth $-z$, σ, and δ are linear in $g(x, y, 0)$. Therefore, there exists a transformation with the following properties: $\sigma = \mathfrak{F}(g)$
if $\sigma_1 = \mathfrak{F}(g_1)$ and $\sigma_2 = \mathfrak{F}(g_2)$, then $\sigma_1 + \sigma_2 = \mathfrak{F}(g_1 + g_2)$; λ is a scalar, then $\lambda\sigma = \mathfrak{F}(\lambda g)$.

This follows from the fact that:

$g(x, y, +z)$ is linear in $g(x, y, 0)$ (see equation(1))

$\dfrac{\partial^{2i} g(x, y, 0)}{\partial z^{2i}}$ is linear in $g(x, y, +z)$ at $z = 0$, thus $g(x, y, -z)$ is linear in $g(x, y, 0)$, and consequently from equation (3) the existence of function $\mathfrak{F}$ is demonstrated.

According to Schwartz's theorem (see Yosida 1968) $\mathfrak{F}$ could be represented by convolution with a kernel function K. We can write

$$\sigma(x_0, y_0, -z) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K(x - x_0, y - y_0, z)\, g(x, y)\, dx dy. \tag{5}$$

From equation (3) it is evident that $\delta(x_0, y_0, z)$ can be expressed by an integral equation similar to (5).

Discretization and limitation of equation (4) gives the equation:

$$\Delta\sigma(i, j, -z, e) = \sum_{k=-M}^{+M} \sum_{k=-N}^{+N} \alpha_{kl}\, g(i - k, j - l) \tag{6}$$

where the α_{kl} depend on the grid dimension, the depth z, and the thickness e.

The value α_{kl} is referred to as the filter coefficient. This denomination follows from a more general idea for the discretization of a convolution integral. For more informations the reader is referred to Ghosh (1971) or Bichara and Lakshmanan (1976).

The idea is the following:

There is theoretical reason for the existence of a filter (α_{kl}) realizing equation (6). This filter has been calculated and tested on various theoretical cases. We found that it was efficient and quite adapted to a density reconstitution at a given depth z.

II – Calculation of the Filter Coefficients

Proposed method

If equation (6) is valid, it follows that the coefficients α_{kl} depend only on geometric factors (grid, depth, and thickness of the source layer). Therefore they are unique and can be computed by considering the effect of any sort of source, for example a right prism.

Let us consider the effect of a right prism of length a_x, height e, situated at a depth z, and with a density contrast of 1, centered on point (m, n) on a grid of dimensions $(2m-1)\,a_x$, $(2n-1)\,a_y$; let $g\,(i, j)$ denote the gravity effect of that prism at point (i, j). The gravity anomaly is given by

$$\Delta g = \frac{2}{3}\Delta\sigma\, e \Big\{ \operatorname{arctg} \frac{(i+1-m)\,(j+1-n)\,a_x a_y}{z\sqrt{(i+1-m)^2\,a_x^2 + (j+1-n)^2 a_y^2 + z^2}}$$

$$+ \operatorname{arctg} \frac{(i-m)\,(j-n)\,a_z a_y}{z\sqrt{(i-m)^2\,ax_x^2 + (j-n)^2 a_y^2 + z^2}}$$

$$- \operatorname{arctg} \frac{(i+1-m)\,(j-)\,n a_x a_y}{z\sqrt{(i+1-m)^2 a_x^2 + (j-n)^2 a_y^2 + z^2}}$$

$$- \operatorname{arctg} \frac{(i-m)\,(j+1-n)\,a_z a_y}{z\sqrt{(i-m)^2 a_x^2 + (j+1-n)^2 a_y^2 + z_2}} \Big\} \qquad (6)$$

Actually, the more accurate equations by Nagy (1966) were used.

We can then write a system of $(m \times n)$ equations

$$\sum_{k=-m}^{+m} \sum_{l=-n}^{+n} \alpha_{kl}\, g\,(i-k, j-l) = 0 \qquad \forall\,(i, j) \neq (m, n) \qquad (7)$$

$$\sum_{k=-m}^{+m} \sum_{l=-n}^{+n} \alpha_{kl}\, g\,(m-k, n-l) = \Delta\sigma, \text{ with the } \alpha_{kl} \qquad (8)$$

as the $m \times n$ unknowns.

Solving these linear equations, we can compute $m \times n$ filter coefficients valid for a given set of values of a_x, a_y, e, and z.

The filter size ($m \times n$) and the grid size $(2m-1) \times (2n-1)$ should in theory be infinitely large. However, we have found empirically that the filter size is correct if outside the complete grid $(2m-1) \times (2n-1)$ the gravity anomaly is less than a hundredth of the maximum anomaly.

II.2 – *Experimental results*

For a right prism with

$a_x = a_y = 10$ m
overburden $z = 10$ m
height $e = 3$ m

density contrast 1 g cm^{-3} we obtain the anomaly shown in fig. 1.

For a grid of dimensions $2m-1 = 2n-1 = 13$ $(m = n = 7)$; the filter coefficients are given on figure 2.

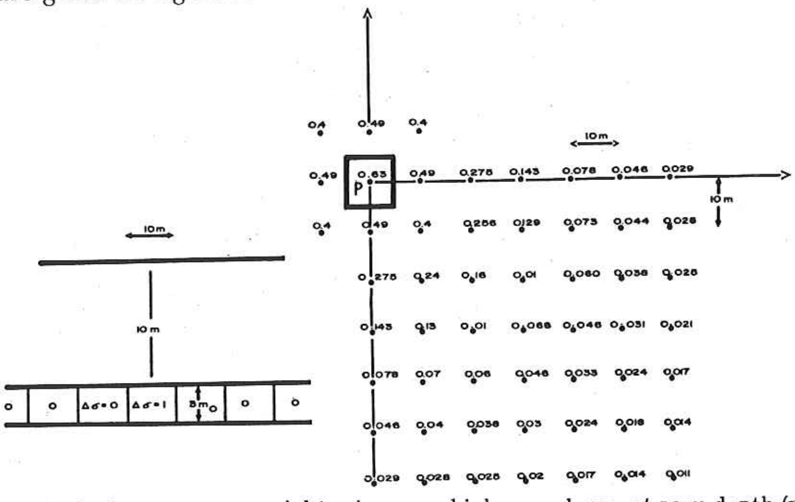

Fig. 1. Anomaly due to a square right prism, 3 m high, 10 m large, at 10 m depth (point P is center of symmetry).

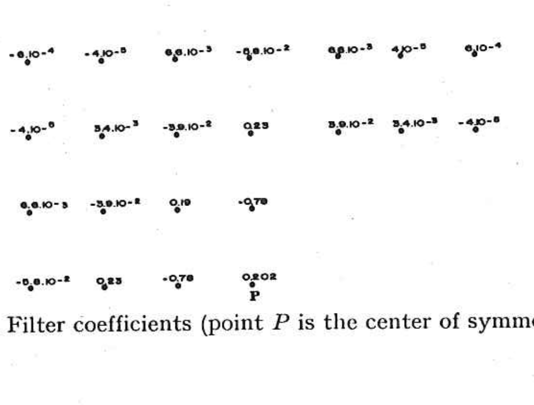

Fig. 2. Filter coefficients (point P is the center of symmetry).

We have

$\sum \alpha_{kl} = 0.07968$ instead of the theoretical value $0.07974 = \frac{3}{4\pi} \cdot \frac{1}{e}$. This theoretical value follows from the equation (4) applied to a layer of constant thickness and contrast:

for $\Delta\sigma = \frac{3\Delta g}{4\pi e}$;

with $e = 3$ m and Δg in 0.01 mgal,

$\Delta\sigma = 0.07974\ \Delta g$. Application of the filter of fig. 2 gives

$\Delta\sigma = (\Sigma\alpha_{kl})\Delta g = 0.07968\Delta g$.

The close agreement confirms elegantly our theoretical assumptions.

III – Experimental results

The following examples concern mathematical control of the accuracy of the filters used. Figure 3 gives the theoretical density distribution: 7 blocks 10 m × 10 m and 3 m height along a line, at 10 m depth with a density contrast of 2 g cm^{-3}. Figure 4 gives the gravity anomaly of this model and figure 5

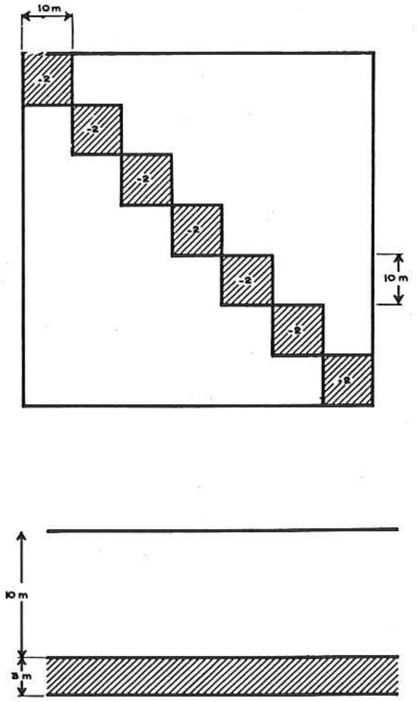

Fig. 3. Model 1.

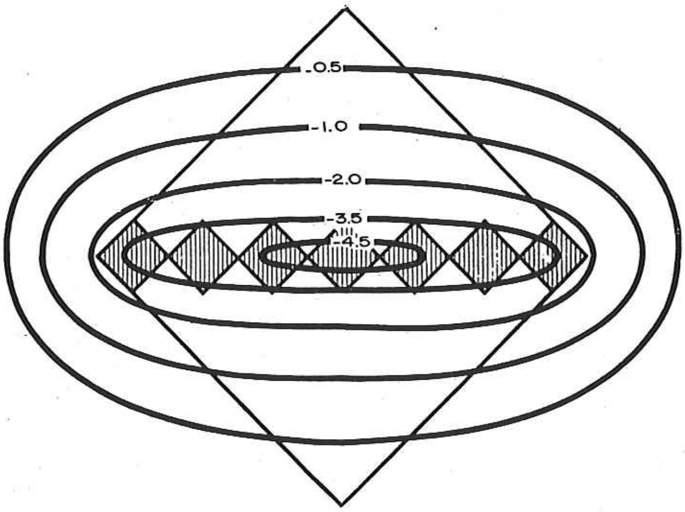

Fig. 4. Anomaly due to model 1 (contours in 0.01 mgal).

-2.005	-.003	-.003	.007	.056	.031	.002
-.003	-2.000	-.000	.001	.021	.066	.031
-.003	-.000	-2.001	-.001	-.000	.021	.056
.007	.001	-001	-2.000	-.001	.001	.007
.056	.021	-.000	-.001	-2.001	-.000	-.003
.031	.066	.021	.001	-.000	-2.000	-.003
.002	.031	.056	.007	-.003	-.003	-2.005

Fig. 5. Model 1 computed by linear filter.
Standard deviation of densities 0.004 g/cm^3.
Standard deviation of recomputed anomaly 0.0006 mGal

gives the reconstructed model. The standard deviation on the density is 0.004 g cm^{-3} (maximum .056). The standard deviation of the gravity anomaly due to this structure compared with the original one is 0·06. 10^{-2} mgal for an average anomaly of 1.60 . 10^{-2} mgal.

The second model is more complicated (figure 6): it consists of 24 little blocks with density contrast -2 g cm^{-2} separated by 25 blocks with density contrast 0.

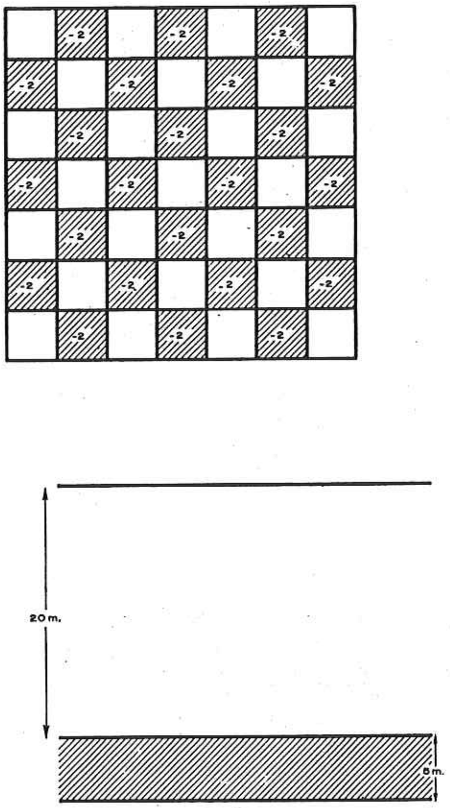

Fig. 6. Model 2.

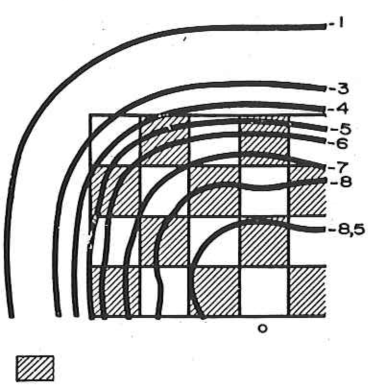

Fig. 7. Anomaly due to model 2 (contours 0.01 mgal).
Point *o* is center of symmetry.

Figure 7 shows a quarter of the anomaly due to this model. The combined effect of the 49 blocks is practically equivalent to that at a single big block with density contrast -1 g cm^{-3}.

The filter result is quite spectacular (figure 8). The mean square difference on the density is .052 g cm^{-3}, the largest difference being. 065 g cm^{-3}. The mean square difference on the reconstructed gravity anomaly is .16 $\cdot 10^{-2}$ mgal for an average of 6.0 $\cdot 10^{-2}$ mgal.

0.16	-1,94	0,027	-1,97	0.027	-1,94	0.016
-1,94	0,07	-1,935	0.030	-1.935	0,087	1 94
0.027	-1.935	0.038	-! 972	0 038	-1,935	0,027
.973	0.03	-1.972	0.0	-1.972	0.03	-1.973
0 .027	-1.936	0.038	-1.972	0,038	-1.935	0,027
-1,94	0,87	-1,935	0,03	-1,935	0,087	-1.94
0.16	-1.94	0,027	-1,97	0,027	-1,94	0,016

Fig. 8. Model 2 computed by linear filter.
Standard deviation of densities 0.025 g/cm^3.
Standard deviation of recomputed anomaly 0.0016 mGal.

Field results

Of course, these quite extraordinary results were obtained on theoretical models, and the reader would be quite justified in asking for practical results. Some of these were presented at the 1974 Hanover Symposium on Engineering Geology and have been controled by drilling. The studies presented at this symposium concerned detection of sinkhole areas in gypsum, 45 m deep.

More recent work has been carried out by CPGF in the Caen area (Normandy, France) where large underground limestone quarries are found. These quarries are approximately 4 m high and have very variable horizontal extension.

Figure 9 shows the map of such a quarry located by a gravity survery, and gives the reconstituted density of prisms 14 m $\times$ 14 m $\times$ 4 m.

The close correlation can be appreciated by computing percentage of quarried surface inside each 14 m $\times$ 14 m square (5) and comparing this percentage with the reconstituted densities D.

Figure 10 gives this correlation. We have $S = -0.385\ (D + 0.75) + 0.383$ or, admitting a least squares relationship passing through the origin $S = -0.465\ D$. This means that the average limestone density contrast for $S = 1$ would be $D = -1/0.465 = -2{,}15$ gcm^{-3}, which seems to be a correct density for this Jurassic limestone.

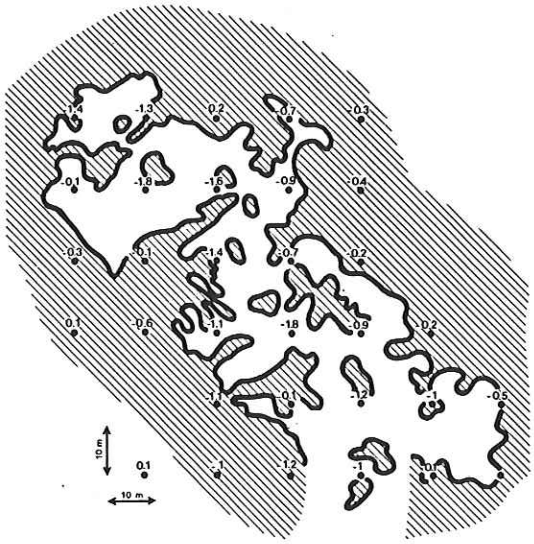

Fig. 9. Comparison between quarry map and densities computed by Filt program.

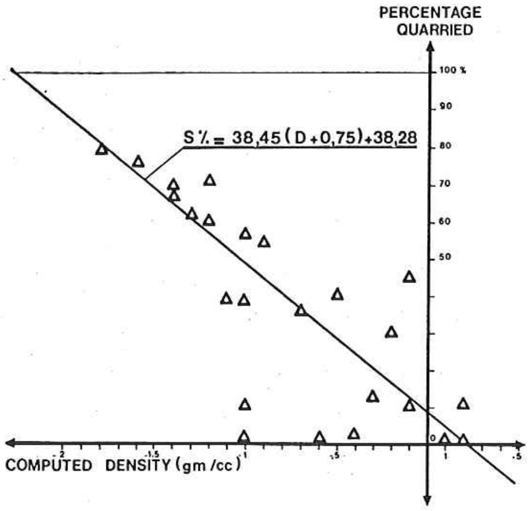

Fig. 10. Correlation between computed density and surface of quarries.

Conclusions

The described work has been carried out by the Compagnie de Prospection Géophysique Française, on a French Government research grant (Délégation Générale à la Recherche Scientifique et Technique). Further work is now being done for systematic use of several filters, to start with a preselected main layer, and building up (or down) density distribution from the residual difference between reconstituted and measured gravity.

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